simpler choice of the possible approaches following Corben (1975, 1976, 1978), in which tachyons and bradyons obey the same fundamental laws of physics, there is no longer any sign ambiguity in the ER transformations.

10. Electromagnetic Four-tensors

Having developed the tachyonic transformations of various four-vectors and electromagnetic quantities, it is now possible to discuss the ER transformations of some electromagnetic four-tensors. The first such tensor is the electromagnetic field tensor $F_{\alpha\beta}$ given by

$$F_{\alpha\beta} = \begin{bmatrix} 0 & B_z & -B_y & -iE_x/c \\ -B_z & 0 & B_x & -iE_y/c \\ B_y & -B_x & 0 & -iE_z/c \\ iE_x/c & iE_y/c & iE_z/c & 0 \end{bmatrix} . \tag{94}$$

The following discussion is adapted from the SR case given by Lawden (1975).

The four-vector potential A_λ in inertial frame Σ is defined to be $A_\lambda = (\mathbf{A}, i\phi/c)$ and so the equations describing the relations between the vector and scalar potentials, (77), (78) and (79), are then equivalent to

$$\square^2 A_\lambda = -\mu_0 J_\lambda \,, \tag{95}$$

where

$$\square^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} \tag{96}$$

and $J_\lambda = (\mathbf{j}, ic\rho)$ is the four-current density. The corresponding equations in tachyonic frame Σ' are

$$\square'^2 A'_\lambda = -\mu_0 J'_\lambda \,, \tag{97}$$

where

$$\square'^2 = \frac{\partial^2}{\partial x'^2} + \frac{\partial^2}{\partial y'^2} + \frac{\partial^2}{\partial z'^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t'^2} \tag{98}$$

and $J'_\lambda = (\mathbf{j}', ic\rho')$. By using A_λ with (70) and (71) it can be shown that the field tensor components may be written as

$$F_{\alpha\beta} = \partial A_\beta / \partial X_\alpha - \partial A_\alpha / \partial X_\beta \,. \tag{99}$$

As the individual components of $F_{\alpha\beta}$ obey the same set of transformations for $u^2 < c^2$ and $u^2 > c^2$ then the field tensor in frame Σ' can be written as

$$F'_{\alpha\beta} = \begin{bmatrix} 0 & B_{z'} & -B_{y'} & -iE_{x'}/c \\ -B_{z'} & 0 & B_{x'} & -iE_{y'}/c \\ B_{y'} & -B_{x'} & 0 & -iE_{z'}/c \\ iE_{x'}/c & iE_{y'}/c & iE_{z'}/c & 0 \end{bmatrix} \tag{100}$$

It can also be shown by using A'_λ with (72) and (73) that the electromagnetic field tensor $F'_{\alpha\beta}$ in tachyonic frame Σ' is

$$F'_{\alpha\beta} = \partial A'_\beta/\partial X'_\alpha - \partial A'_\alpha/\partial X'_\beta\,. \tag{101}$$

By substituting the tachyonic transformations for the partial derivatives (18) and electromagnetic potentials (74) into (101) it can be shown that

$$F'_{\alpha\beta} = \begin{bmatrix} 0 & \gamma_u(B_z - uE_y/c^2) & -\gamma_u(B_y + uE_z/c^2) & -iE_x/c \\ -\gamma_u(B_z - uE_y/c^2) & 0 & B_x & -i\gamma_u(E_y - uB_z)/c \\ \gamma_u(B_y + uE_z/c^2) & -B_x & 0 & -i\gamma_u(E_z + uB_y)/c \\ iE_x/c & i\gamma_u(E_y - uB_z)/c & i\gamma_u(E_z + uB_y)/c & 0 \end{bmatrix}.$$

This same result could have been obtained simply by transforming the individual tensor components according to (21) and (22), which are valid for $-\infty < u < \infty$. Hence for $u^2 > c^2$ the electromagnetic field tensor transforms according to

$$F'_{\alpha\beta} = \sum_{\mu=1}^{4}\sum_{\nu=1}^{4} L_{\alpha\mu}F_{\mu\nu}L'_{\nu\beta}\,. \tag{102}$$

As this is exactly the same transformation as for $u^2 < c^2$, then (102) is valid for $-\infty < u < \infty$. The tachyonic four-vectors involved in this transformation are A_λ and D_λ which transform according to the upper and lower signs respectively in (90), so the factors of $+i$ and $-i$ in the ER transformations of these quantities will combine to give $+1$ for the field tensor. This again demonstrates how the imaginary factors in this formulation cancel out when appropriate to produce reasonable and consistent results.

The components of the electromagnetic stress–energy tensor $T_{\alpha\beta}$ are given by (Muirhead 1973; Jackson 1975; Lawden 1975)

$$T_{ij} = \delta_{ij}(\epsilon_0 E^2 + B^2/\mu_0)/2 - (\epsilon_0 E_i E_j + B_i B_j/\mu_0)\,, \tag{103}$$

$$T_{j4} = T_{4j} = i(\mathbf{E}\times\mathbf{B})_j/c\mu_0 = i\mathbf{S}_j/c\,, \tag{104}$$

$$T_{44} = -(\epsilon_0 E^2 + B^2/\mu_0)/2 = -U_{em}\,, \tag{105}$$

where $\mathbf{S}$ is Poynting's vector and U_{em} is the energy density of the electromagnetic field. As the components of $T_{\alpha\beta}$ undergo the same transformations for $u^2 > c^2$

as they do for $u^2 < c^2$, then $T'_{\mu\nu}$ must be related to $T_{\alpha\beta}$ by

$$T'_{\mu\nu} = \sum_{\alpha=1}^{4} \sum_{\beta=1}^{4} L_{\mu\alpha} T_{\alpha\beta} L'_{\beta\nu} \tag{106}$$

for $-\infty < u < \infty$. Further electromagnetic four-tensors involving electric displacement and magnetic field strength will be discussed in the next paper in this series (Paper IV), which will examine further aspects of electrodynamics for tachyons.

11. Conclusion

It has now been shown that charged tachyons can be incorporated into the theory of electromagnetism in a logical and consistent manner. Tachyons obey Maxwell's equations in free space, as required by the first postulate of ER in which 'the laws of physics are the same in all inertial systems'. The imaginary factors and high relative speeds appearing in the theory of tachyons do not change the transformations of **E** and **B**, the form of the Lorentz force law or the transformations of the electromagnetic field tensor or stress–energy tensor.

The total electric charge in any given inertial reference frame is always conserved, but the apparent charge is no longer the same when measured by different observers due to some of the tachyons appearing to be switched in some frames. This does not present a problem, as it is always possible to transform to another reference frame in which all of the tachyons appear to be unswitched. Then the total charge is the same as if all the particles were bradyons instead.

Fundamental constants such as the permittivity and permeability of free space have been shown to be the same regardless of whether the observer's inertial reference frame is bradyonic or tachyonic. This is a consequence of the two postulates of ER, but the rigorous derivation of this result serves to demonstrate the internal consistency of this formulation of tachyon theory. This point is significant for other branches of physics, not just for electromagnetism. In quantum mechanics it can be safely assumed that Planck's constant is invariant for bradyons and tachyons, thus simplifying some of the derivations for quantum tachyons which have been adapted from the corresponding relativistic cases (Dawe 1990). Dawe *et al.* (1989) have used an invariant Boltzmann constant when developing statistical mechanics and thermodynamics for tachyons.

The electric and magnetic fields (or alternatively the scalar and vector potentials) produced by a charged tachyon travelling through a vacuum at constant velocity are real and in principle detectable inside a Mach cone having semivertex angle $|c/u| = |\sin\theta|$, in agreement with Barut *et al.* (1982) and Recami (1986) and references therein. As the field is real and moves with constant speed c regardless of the source speed, any point lying outside the cone corresponds to a position where the field is purely imaginary and is therefore undetectable. At the instant of contact with the cone describing the propagation of the field, any detection instruments would register a sudden jump called an 'optic boom', in analogy with the 'sonic boom' generated by supersonic aircraft (Barut *et al.* 1982). After the instant of initial contact, the image of the tachyon splits into two images travelling in opposite directions, an effect called the 'two source effect' (Recami *et al.* 1986; Recami 1986 and references therein). One of the images represents the tachyon as it travels forwards, while the second image which appears to go backwards is due purely to the time delay associated with electromagnetic effects having a fixed and finite speed, even though this is exceeded by the source speed. As neither the optic boom or the two source effect can be produced by individual particles which appear to be bradyons relative to the observer, then these effects would constitute definitive evidence of the existence of tachyons should they be detected in the laboratory.

The various tachyonic transformations were also shown to be consistent with the expected Doppler effects for relative approach (blueshift) and recession (redshift), including predicting the order in which signals from the tachyon would be received. The transverse Doppler effect for tachyons is a blueshift for $u^2 > 2c^2$ and a redshift for $u^2 < 2c^2$.

The overall result of the work presented in this paper is to demonstrate that charged tachyons, if they exist, can obey Maxwell's equations in a vacuum. In analogy with ordinary relativistic particles, tachyons have been shown to possess real and in principle detectable attributes such as an electromagnetic field and a Doppler effect. Moreover, in their own inertial reference frame, tachyons behave like bradyons and a comoving observer would consider them to be travelling more slowly than the propagation speed of electromagnetic radiation in a vacuum.

In the light of the above results for tachyonic electrodynamics in vacuo, further developments become possible and will be investigated in the next paper of the present series. The extension of the present work to cover tachyonic electrodynamics in a medium will be undertaken with the consideration of the electric displacement and polarisation vectors, together with the magnetic field intensity and magnetisation vectors. The treatment of these topics will be followed up by discussions of the electric dipole moment of a tachyonic current loop, constitutive equations and the velocity of light in a tachyonic medium. In order to provide a comprehensive picture of the behaviour of tachyons at the classical relativistic level as well as preparing the ground for tachyonic quantum mechanics, Paper IV will also contain a discussion of Lagrange's and Hamilton's equations for charged tachyons. There will also be an explanation of why tachyons can be considered to be effectively localised particles.

As foreshadowed in the Conclusion to Paper II, Paper IV will also consider the question of radiation emission by charged tachyons. This is a topic which is to be regarded as of critical importance in determining the existence or otherwise of tachyons. As pointed out by Treumann (1992), if tachyons can emit bremsstrahlung in collisions with other particles, regardless of whether the other particles are bradyonic or tachyonic, and also emit synchrotron radiation in interaction with a magnetic field, then this radiation should ultimately be detected by astrophysical observation. If such radiation is not detected then a choice must be made between the following alternatives: (i) tachyons do not exist, (ii) tachyons may exist but must be uncharged, or (iii) charged tachyons exist but radiate with a spectrum which is undetectable: this last option seems unphysical. Thus the study of electrodynamics for tachyons could lead to astrophysical observations to determine if any of these three options is appropriate, or if tachyons do in fact exist. In Paper IV a beginning is made with a treatment of Cerenkov radiation for tachyonic particles in tachyonic media.

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