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The Physics of Tachyons
II. Tachyon Dynamics*

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Abstract

This paper extends the development of a new formulation of the theory of tachyons to encompass tachyon dynamics. Topics discussed include tachyonic energy and momentum, proper mass, force and acceleration.

1. Introduction

The aim of this paper is to build upon the work of Dawe and Hines (1992) (hereafter referred to as Paper I) in developing a new formulation of the theory of tachyons which is logical and self-consistent. The material in this paper extends the theory of tachyons to cover the subject of dynamics and discusses topics such as energy, momentum, proper mass, force and acceleration.

The overall result of Paper I was to show that the framework of special relativity (SR) can be extended to include particles having a relative speed greater than the speed of light in free space. The requirements necessary to allow this extension of special relativity into extended relativity (ER) are the switching principle (expressed mathematically as the 'γ-rule'), a standard convention for decomposing imaginary square roots and the minor modification of some familiar definitions such as 'source' and 'detector'. The results and definitions of ER automatically reduce to those of SR as soon as the objects appear to the observer to be bradyons.

Some terms will be in common usage throughout this work, so they will be defined here. 'Special relativity' (SR) refers to all currently accepted physics for particles which travel more slowly relative to the observer than the speed of light. These particles will henceforth be called 'bradyons'. A 'tachyon' is defined to be a particle which is travelling relative to the observer at a speed greater than the speed of light. 'Extended relativity' (ER) is the theoretical framework which describes the motion and interactions of tachyons. A 'bradyonic observer' travels at a speed less than c, while a 'tachyonic observer' travels at a speed greater than c relative to the inertial reference frame of the laboratory.

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The theory being developed in this series of papers is founded upon the philosophy of Corben (1978) who has argued that tachyons, should they exist, 'are basically the same objects as ordinary particles (they just look different because they are moving so fast).' With this in mind, the same two postulates apply in ER as in SR:

Postulate 1: The laws of physics are the same in all inertial systems.

Postulate 2: The speed of light in free space has the same value c in all inertial systems.

The term 'inertial system' now refers to any system travelling at a constant velocity with respect to an inertial observer, irrespective of whether the system is travelling slower than or faster than the speed of light. The postulates lead to the Lorentz transformations if the relative speed u between the two inertial reference frames is such that $u^2 < c^2$, but to the following superluminal Lorentz transformations (SLTs) if the relative speed between the two inertial reference frames is such that $u^2 > c^2$:

$$x' = i\gamma_u(x - ut), \;\; y' = iy, \;\; z' = iz, \;\; t' = i\gamma_u(t - ux/c^2)\,. \qquad (1)$$

Here u is the relative speed along the common x, x' axes of an inertial reference frame Σ' with respect to an inertial reference frame Σ and

$$\gamma_u = (1 - u^2/c^2)^{-1/2} \qquad (2)$$

for both $u^2 < c^2$ and $u^2 > c^2$.

When $u^2 > c^2$ the inertial frames Σ and Σ' are on opposite sides of the light barrier and a particle seen by Σ as a bradyon would be seen as a tachyon by Σ' and vice versa. Even though tachyonic transformations such as (1) indicate that the axes transverse to the boost are imaginary while the axis parallel to the boost remains real, an inertial observer in the rest frame of the tachyon considers all of the axes to be real.

In Paper I it was shown that tachyons can logically and consistently obey the conservation laws of energy, momentum and electric charge through the use of a 'switching principle' (expressed mathematically as the 'γ-rule'). A detailed numerical example was used to demonstrate that *'for switched tachyons the negative root of γ_u is used and for unswitched tachyons the positive root of γ_u is used.'* Written explicitly, this rule is

$$\gamma_u = \text{sign}(\gamma_u)(1 - u^2/c^2)^{-1/2} \qquad (3)$$

where $\text{sign}(\gamma_u) = +1$ if the particle appears to an observer to be an unswitched tachyon or a bradyon, and $\text{sign}(\gamma_u) = -1$ if the particle appears to that observer to be a switched tachyon. Note that as there is no switching for a particle viewed by an observer to be a bradyon, then $\text{sign}(\gamma_u)$ is always $+1$ and the standard result of SR is automatically recovered. Here the speed u is the relative speed between two inertial reference frames and should not be confused with the speed of the particle in the observer's inertial reference frame.

The tachyonic velocity transformations, which are exactly the same in ER and SR, automatically show whether the particle is switched or unswitched relative to a particular observer. Let v_x be the speed of the particle in the initial frame

Σ, while u is the speed of the final frame Σ' relative to Σ along the common x, x' axes. The particle will appear to Σ' to be switched if

$$c > u > c^2/v_x \text{ for } v_x > c \text{ and } |u| < c, \text{ or} \tag{4}$$

$$c < u < c^2/v_x \text{ for } v_x < c \text{ and } |u| > c. \tag{5}$$

To be consistent in the calculations of ER, a convention is used to deal with imaginary square roots such that when $u^2 > c^2$ then

$$(1 - u^2/c^2)^{1/2} = i(u^2/c^2 - 1)^{1/2}. \tag{6}$$

In Paper I it was also shown that tachyonic rods exhibit contraction and dilation effects and that tachyonic clocks appear to be slowed down or to run fast depending on the speed relative to the observer. A further example considered the visual appearance of a tachyonic cube. In all of these examples the tachyonic object demonstrated logical and consistent behaviour within the theory.

In this paper the theory is extended to cover the dynamics of tachyons. Energy and momentum are discussed in Section 2, along with the proper mass of a tachyon. The transformation of force in ER is derived in Section 3. In Section 4 acceleration in ER is discussed, as well as the relationship between force and acceleration. Two simple examples relating to the motion of a charged tachyon are discussed, followed by a brief explanation of why tachyons cannot emit Cerenkov radiation in a vacuum.

The metric $(+1, +1, +1, +1)$ is used throughout this work, with the convention that Greek indices λ, μ, ν, ... run from 1 to 4. The position four-vector is written as $X_\lambda = (x_1, x_2, x_3, x_4) = (\mathbf{x}, ict) = (x, y, z, ict)$. Note that the metric $(+1, +1, +1, +1)$ applies to both bradyonic and tachyonic inertial reference frames, as the metric is independent of the observer's relative motion and is the same regardless of whether the observer is dealing with bradyons or tachyons. Furthermore, there is no distinction made in the following work between covariant and contravariant quantities. For a discussion of the reasons behind these choices the reader is referred to Paper I.

2. Energy, Momentum and Mass

To derive the energy–momentum relations in ER, it is first necessary to have an appropriate Lagrangian for each class of particle. The Lagrangian for a bradyon in the absence of external fields or potentials is

$$L_B = -m_o c^2 (1 - v^2/c^2)^{1/2}, \qquad v^2 < c^2, \tag{7}$$

where m_o is the proper mass of the particle. In Paper I it was shown that expressions for tachyons generally have the same form as for bradyons, but with extra factors of i or $-i$ included as necessary. As the square root in the γ-factor has an identical form for bradyons and tachyons, it is therefore convenient to try a Lagrangian for a free tachyon of the form

$$L_T = -m_* c^2 (1 - v^2/c^2)^{1/2}, \qquad v^2 > c^2, \tag{8}$$

where m_* is the proper mass of the tachyon. Note that L_T is the same as in the Recami formulation (1986) except for the possible intrinsic sign change in the square root due to apparent switching in some reference frames. A Lagrangian for charged tachyons in an electromagnetic potential will be discussed in a later paper.

For bradyons the momentum $\mathbf{p}_B$ is defined as

$$\mathbf{p}_B = \partial L_B / \partial \mathbf{v} = \gamma_v m_o \mathbf{v}\,, \tag{9}$$

where $\mathbf{v}$ is the velocity of the particle. The momentum $\mathbf{p}_T$ for tachyons is defined in exactly the same way:

$$\mathbf{p}_T = \partial L_T / \partial \mathbf{v} = \gamma_v m_* \mathbf{v}\,. \tag{10}$$

The total energy for bradyons E_B is defined as

$$E_B = \mathbf{p_B}.\mathbf{v} - L_B = \gamma_v m_o c^2 \tag{11}$$

and for tachyons the total energy E_T is defined as

$$E_T = \mathbf{p_T}.\mathbf{v} - L_T = \gamma_v m_* c^2\,. \tag{12}$$

If tachyons are to obey the laws of physics, as required by the first postulate of ER, then they must be able to emit photons. As photons have positive real energy and γ_v is imaginary for $v^2 > c^2$, then (12) shows that the tachyon's proper mass m_* must be imaginary. In the ultrarelativistic limit as $v \to c$ the properties of a tachyon become identical to the properties of a bradyon in the same limit, so that m_* must have the same magnitude as m_o. The tachyon's effective mass $\gamma_v m_*$ is positive and real in frames in which the tachyon appears to be unswitched, and so the convention used for dealing with imaginary square roots given by (6) means that m_* must be a positive imaginary quantity. Hence m_* is related to the bradyonic proper mass m_o via the relation $m_* = i m_o$.

Combining (10) with (12) leads to the energy–momentum relation for tachyons:

$$E_T = \gamma_v m_* c^2 = (p^2 c^2 + m_*^2 c^4)^{1/2}\,. \tag{13}$$

Thus the complete set of energy–momentum relations for all relative speeds v is, with $m_* = i m_o$ for tachyons,

$$E^2 = \begin{cases} p^2c^2 + m_o^2 c^4 & \text{for bradyons, } v^2 < c^2 \\ p^2 c^2 & \text{for photons, } v^2 = c^2 \\ p^2c^2 - m_o^2 c^4 & \text{for tachyons, } v^2 > c^2\,. \end{cases} \tag{14}$$

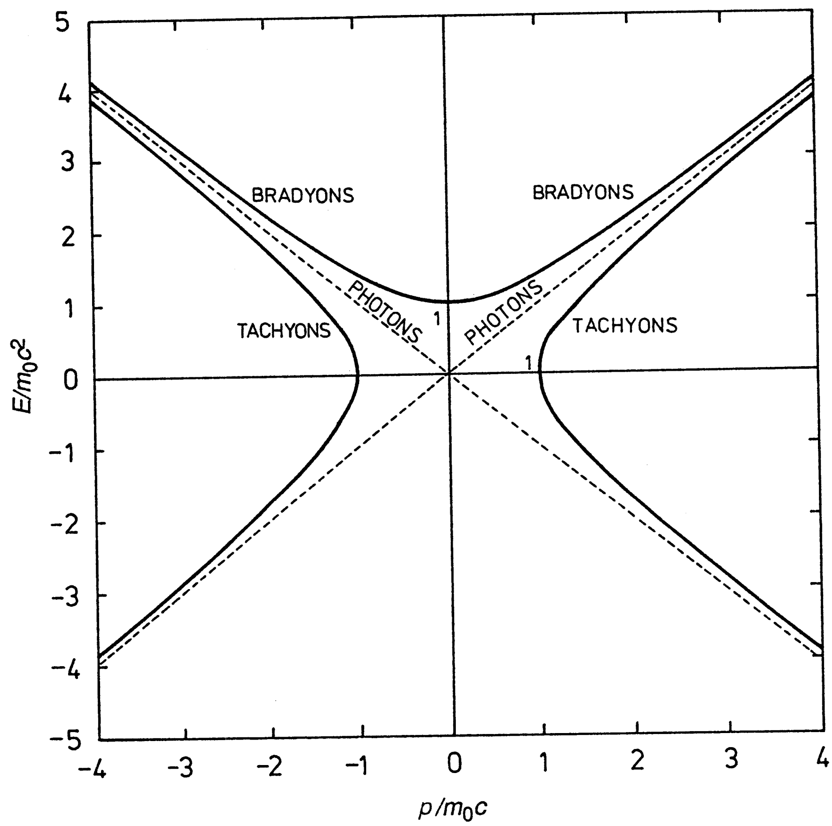

Fig. 1. A plot of energy against momentum in reduced units for bradyons, photons and tachyons.

The three cases in (14) have been plotted in Fig. 1. For bradyons the energy is a minimum at m_oc^2 when $p = 0$, whereas tachyons can have any energy but have a limit on momentum such that $|p| \geq m_oc$. Both bradyons and tachyons approach the photon asymptotes at high energies and momenta.

Fig. 2 is a plot of energy against speed for bradyons, photons and tachyons. For both bradyons and tachyons the speed approaches c as their energy increases. Note that a tachyon will be accelerated to higher speeds as it loses energy, but cannot end up with a negative energy via steady acceleration: the limit is $E = 0$. The curve describing nonrelativistic bradyons (i.e. 'Newtonian' particles) crosses the tachyonic curve at $v/c = \pm(2)^{1/2}$: this is the 'Newtonian limit' for tachyons.

Fig. 3 is a plot of momentum against speed for bradyons, photons and tachyons. It clearly shows that as bradyons gain speed they gain momentum, whereas tachyons lose momentum as they gain speed. Tachyons have an asymptotic momentum limit of $\pm m_oc$ as $v/c \to \pm\infty$: this corresponds to $E \to 0$. Tachyons also have $p \to \pm\infty$ as $v/c \to \pm 1$, just like bradyons. Note that Figs 1, 2 and 3 are in agreement with the work of Bilaniuk and Sudarshan (1969).

In special relativity the relative speed u between two inertial reference frames can be steadily decreased to arrive at the standard nonrelativistic limit as $u \to 0$. In this case the usual Newtonian results are recovered, such as rods having

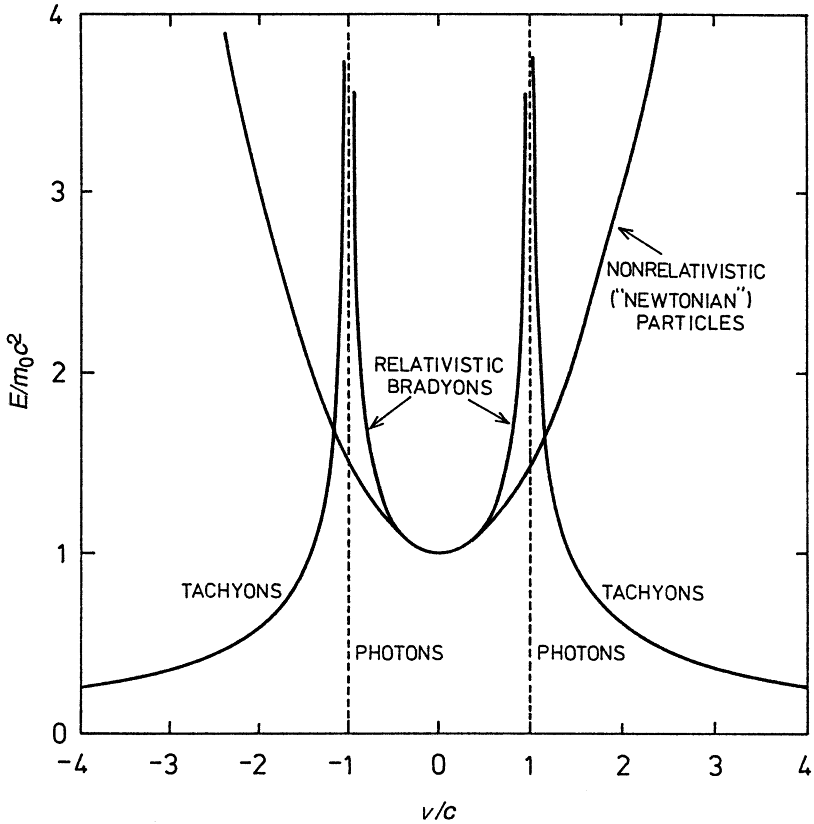

Fig. 2. A plot of energy against speed in reduced units for nonrelativistic and relativistic particles as well as photons and tachyons.

the same length in different reference frames, clocks running at the same rate, momentum being given by $\mathbf{p} = m_o\mathbf{u}$ and the total energy of a freely moving body being given by $E = \gamma_u m_o c^2 \approx m_o c^2 + m_o u^2/2$. In each case the nonrelativistic result is obtained because $\gamma_u \to 1$ as $u \to 0$. For tachyons it can be seen that $|\gamma_u| \to 1$ as $u \to \pm c(2)^{1/2}$. If the particle speed u is exactly $\pm c(2)^{1/2}$ then tachyonic rods do not appear to be contracted or dilated and tachyonic clocks appear to run at the same rate. This speed is the 'Newtonian limit' for tachyons and is the border between various contraction and dilation effects, as was shown in Paper I.

In bradyonic inertial reference frame Σ the momentum and energy of a bradyon are given by $\mathbf{p} = \gamma_v m_o \mathbf{v}$ and $E = \gamma_v m_o c^2$ respectively, where $\mathbf{v}$ is the relative velocity of the particle. Now suppose there is an inertial reference frame Σ' moving with speed $u > c$ along the common x, x' axes relative to frame Σ. In tachyonic frame Σ' the particle appears to be a tachyon with velocity $\mathbf{v}'$ and with momentum and energy given by $\mathbf{p}' = \gamma_{v'} m_* \mathbf{v}'$ and $E' = \gamma_{v'} m_* c^2$ respectively. The particle has proper mass m_o when it appears to the observer as a bradyon, but has proper mass m_* when it appears to the observer as a tachyon.

To derive the ER energy–momentum transformations some results from Paper I are needed. The velocity transformations from frame Σ' to frame Σ, which are

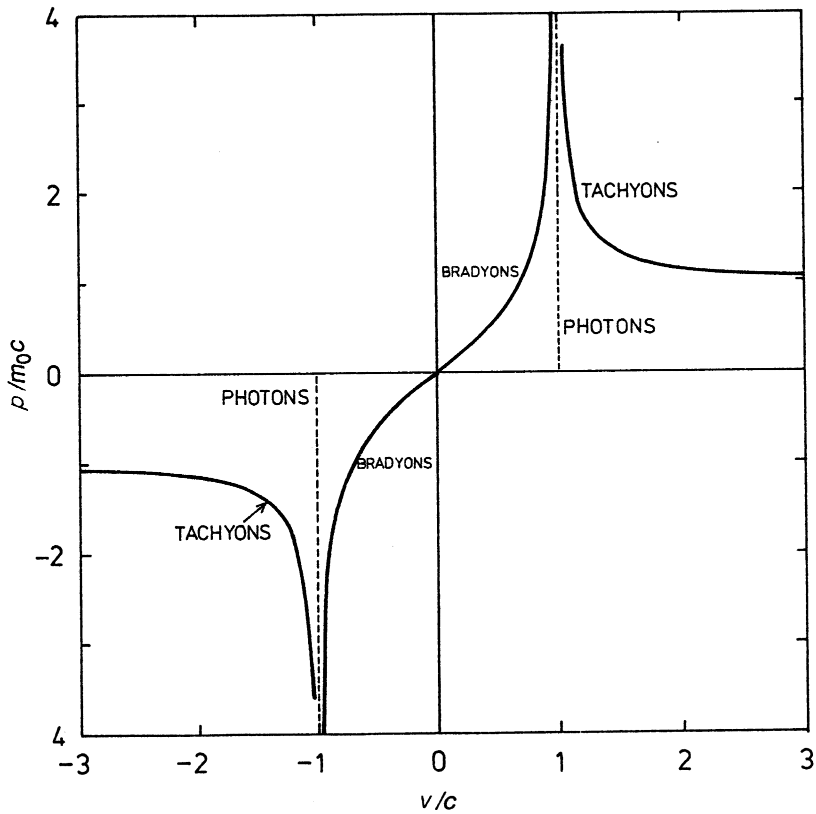

Fig. 3. A plot of momentum against speed in reduced units for bradyons, photons and tachyons.

valid for $-\infty < u < \infty$, are

$$v_x = \frac{v_{x'} + u}{1 + uv_{x'}/c^2}, \quad v_y = \frac{v_{y'}}{\gamma_u(1 + uv_{x'}/c^2)}, \quad v_z = \frac{v_{z'}}{\gamma_u(1 + uv_{x'}/c^2)}. \tag{15}$$

The velocity transformations from Σ to Σ' are

$$v_{x'} = \frac{v_x - u}{1 - uv_x/c^2}, \quad v_{y'} = \frac{v_y}{\gamma_u(1 - uv_x/c^2)}, \quad v_{z'} = \frac{v_z}{\gamma_u(1 - uv_x/c^2)}. \tag{16}$$

The velocity transformations lead to the following useful equalities, which are also valid for $-\infty < u < \infty$:

$$\gamma_v = \gamma_u \gamma_{v'}(1 + uv_{x'}/c^2), \tag{17}$$

$$\gamma_{v'} = \gamma_u \gamma_v(1 - uv_x/c^2). \tag{18}$$

Using these results and $m_* = im_o$ in the energy and momentum expressions in frame Σ leads to the inverse tachyonic energy–momentum transformations:

$$p_x = -i\gamma_u(p_{x'} + uE'/c^2),\ \ p_y = -ip_{y'},\ \ p_z = -ip_{z'},\ \ E = -i\gamma_u(E' + up_{x'})\,. \quad (19)$$

To obtain the energy–momentum transformations as seen by Σ' the following general rules can be used:

(1) Interchange primed and unprimed quantities.

(2) Reverse the sign of u.

(3) Reverse the sign of i.

(4) Replace m_* with im_o if either proper mass appears in the transformation expression and the two frames are on opposite sides of the light barrier (i.e. one frame is bradyonic while the other is tachyonic).

The first two of these rules for transforming quantities between frames Σ and Σ' are the same ones as those used in special relativity. The last two rules represent the extension into ER with its intrinsic imaginary quantities. These rules apply to all quantities that have transformations in ER.

In the present example of energy–momentum transformations, using these transformation rules gives

$$p_{x'} = i\gamma_u(p_x - uE/c^2)\,,\quad p_{y'} = ip_y\,,\quad p_{z'} = ip_z\,,\quad E' = i\gamma_u(E - up_x)\,. \quad (20)$$

The transformations given by (20) can easily be verified by substitution of relevant quantities. These expressions are similar to those given by Recami (1986), except that he has a factor of ±1 multiplying the transverse components and a factor of ∓1 multiplying the $p_{x'}$ and E' components.

The corresponding relativistic transformations of momentum and energy have the same form as (20) but without the factor of i in each component, and so the momentum four-vector $P_\lambda = (\mathbf{p}, iE/c)$ transforms in the same way as the position four-vector $X_\lambda = (\mathbf{x}, ict)$ discussed in Paper I. Hence the square of the energy–momentum four-vector is

$$E'^2/c^2 - p_{x'}^2 - p_{y'}^2 - p_{z'}^2 = \pm(E^2/c^2 - p_x^2 - p_y^2 - p_z^2)\,, \quad (21)$$

where the upper sign applies to transformations between inertial reference frames on the same side of the light barrier (bradyonic to bradyonic or tachyonic to tachyonic) and the lower sign applies to transformations between inertial reference frames on opposite sides of the light barrier (bradyonic to tachyonic and vice versa).

The 'relativistic' mass of a bradyon is given by $m = \gamma_v m_o$ in bradyonic frame Σ and by $m' = \gamma_{v'} m_*$ in tachyonic frame Σ'. Using $m_* = im_o$ with (17) and (18) leads to the tachyonic transformation of mass between frames Σ and Σ' as

$$m' = im\gamma_u(1 - uv_x/c^2)\,, \quad (22)$$

$$m = -im'\gamma_u(1 + uv_{x'}/c^2)\,. \quad (23)$$

The equivalent transformations in SR (Rosser 1964) can be obtained from the ER expressions by removing the factors of i and $-i$ as appropriate.

3. Transformation of Force

The next stage in the development of dynamics for tachyons is to derive the force transformations. This will be done using a method similar to that given by Rosser (1960) in the relativistic case. The concept of a force acting on a tachyon is valid as the particle is a bradyon at rest in its own reference frame, and is therefore able to interact with any forces or potentials such as those generated via mechanical interaction, electrodynamics or gravitation.

Consider an inertial reference frame Σ, relative to which a particle T appears to be a bradyon with velocity $\mathbf{v}$ such that $v^2 < c^2$. In this frame the components of the force acting on the particle are

$$F_x = m_o \frac{d}{dt}(\gamma_v v_x)\,, \quad F_y = m_o \frac{d}{dt}(\gamma_v v_y)\,, \quad F_z = m_o \frac{d}{dt}(\gamma_v v_z) \tag{24}$$

where $\gamma_v = (1 - v^2/c^2)^{-1/2}$. A second inertial reference frame Σ' moves along the common x, x' axes with speed $u > c$ relative to frame Σ. In frame Σ' the particle T appears to be a tachyon with velocity $\mathbf{v}'$ such that $v'^2 > c^2$. In Σ' the particle experiences the components of the force as

$$F_{x'} = m_* \frac{d}{dt'}(\gamma_{v'} v_{x'})\,, \quad F_{y'} = m_* \frac{d}{dt'}(\gamma_{v'} v_{y'})\,, \quad F_{z'} = m_* \frac{d}{dt'}(\gamma_{v'} v_{z'}) \tag{25}$$

where $\gamma_{v'} = (1 - v'^2/c^2)^{-1/2}$.

Using the expressions $\gamma_v v_x = \gamma_u \gamma_{v'}(u + v_{x'})$ and $dt/dt' = -i\gamma_u(1 + uv_{x'}/c^2)$ with $m_* = im_o$ gives

$$F_x = \frac{m_*}{1 + uv_{x'}/c^2}\left(\frac{d}{dt'}(\gamma_{v'} v_{x'}) + u\frac{d}{dt'}(\gamma_{v'})\right). \tag{26}$$

Adding and subtracting the term $\{m_* uv_{x'}/(c^2 + uv_{x'})\}(d/dt')(\gamma_{v'} v_{x'})$ to the right hand side of (26) and using the relations

$$1 - v_{x'}^2/c^2 = \gamma_{v'}^{-2} + v_{y'}^2/c^2 + v_{z'}^2/c^2 \tag{27}$$

and

$$\frac{d\gamma_{v'}}{dt'} = \left(v_{x'}\frac{dv_{x'}}{dt'} + v_{y'}\frac{dv_{y'}}{dt'} + v_{z'}\frac{dv_{z'}}{dt'}\right)\frac{\gamma_{v'}^3}{c^2} \tag{28}$$

to simplify terms leads to the tachyonic transformation of F_x:

$$F_x = F_{x'} + (v_{y'}F_{y'} + v_{z'}F_{z'})u/(c^2 + uv_{x'})\,. \tag{29}$$

Using the identities $\gamma_v v_y = \gamma_{v'} v_{y'}$ and $\gamma_v v_z = \gamma_{v'} v_{z'}$ leads to the transformation of the y and z components of the force:

$$F_y = \frac{F_{y'}}{\gamma_u(1 + uv_{x'}/c^2)}, \quad F_z = \frac{F_{z'}}{\gamma_u(1 + uv_{x'}/c^2)}\,. \tag{30}$$

The inverses of these tachyonic force transformations are

$$F_{x'} = F_x - \frac{(v_y F_y + v_z F_z)u}{c^2 - uv_x}, \quad F_{y'} = \frac{F_y}{\gamma_u(1 - uv_x/c^2)}, \quad F_{z'} = \frac{F_z}{\gamma_u(1 - uv_x/c^2)}. \tag{31}$$

The tachyonic force transformations have the same form as in the relativistic case, so they are valid for $-\infty < u < \infty$. Note that $F_{x'}$ is always real, while $F_{y'}$ and $F_{z'}$ are real for $u^2 < c^2$ and imaginary for $u^2 > c^2$, in agreement with the worldview of tachyons discussed in Paper I. The transverse components of the force disappear in the limit $u \to c^{\pm}$. When $uv_x = c^2$ the apparent force is instantaneously infinite, but this is just another manifestation of the dual speed condition and represents the transition between inertial reference frames in which the tachyon is unswitched and those frames in which the tachyon appears to the observer to be switched.

4. Acceleration in Extended Relativity

Acceleration Transformations

In special relativity all the frames of reference used by observers are taken to travel at a constant velocity, indeed many SR texts stress the importance of not having any observer undergo acceleration. However, it is still meaningful to discuss the acceleration of objects as viewed by inertial observers, especially for the cases of charged particles in electromagnetic fields.

The acceleration in inertial frame Σ' is defined to be

$$\mathbf{a}' = d\mathbf{v}'/dt' = (d\mathbf{v}'/dt)(dt/dt') = -i\gamma_u(1 + uv_{x'}/c^2)d\mathbf{v}'/dt \tag{32}$$

using the tachyonic transformation of t for the last step. Substituting the various components of the tachyonic velocity transformations into (32) leads to the tachyonic acceleration transformations:

$$a_{x'} = \frac{-ia_x}{\gamma_u^3(1 - uv_x/c^2)^3}, \quad a_{y'} = \frac{-i\left(a_y + uv_y a_x/(c^2 - uv_x)\right)}{\gamma_u^2(1 - uv_x/c^2)^2},$$

$$a_{z'} = \frac{-i\left(a_z + uv_z a_x/(c^2 - uv_x)\right)}{\gamma_u^2(1 - uv_x/c^2)^2}. \tag{33}$$

The inverse tachyonic acceleration transformations can be immediately written using the rules (i) interchange primed and unprimed variables, (ii) reverse the sign of u and (iii) reverse the sign of i. (The additional transformation rule involving the proper mass is irrelevant for acceleration.) This procedure gives

$$a_x = \frac{ia_{x'}}{\gamma_u^3(1 + uv_{x'}/c^2)^3}, \quad a_y = \frac{i\left(a_{y'} - uv_{y'} a_{x'}/(c^2 + uv_{x'})\right)}{\gamma_u^2(1 + uv_{x'}/c^2)^2},$$

$$a_z = \frac{i\left(a_{z'} - uv_{z'} a_{x'}/(c^2 + uv_{x'})\right)}{\gamma_u^2(1 + uv_{x'}/c^2)^2} \tag{34}$$

which can be confirmed by substitution of appropriate quantities.

Again it can be seen that for tachyonic frames the longitudinal component $a_{x'}$ is real while the transverse components $a_{y'}$ and $a_{z'}$ are imaginary. The corresponding transformations in SR, given for example by Rosser (1964), can be obtained by removing the $-i$ in (33) and the i in (34).

Relationship between Force and Acceleration in ER

In SR the relationship between force and acceleration is no longer given by the simple Newtonian expression $\mathbf{F} = m\mathbf{a}$ and the same is true in ER. What follows is a derivation of the relationship between $\mathbf{F}$ and $\mathbf{a}$ applicable to both SR and ER.

Consider the work done on a particle by a constant force $\mathbf{F}$ in moving that particle through a displacement $d\mathbf{l}$. In both Newtonian and relativistic mechanics the work done is defined to be $dW = \mathbf{F}.d\mathbf{l}$. If it is assumed that all of the work goes into increasing the kinetic energy K of the particle then $dW = dK$. Hence the rate of increase of kinetic energy is given by $dK/dt = \mathbf{F}.d\mathbf{l}/dt = \mathbf{F}.\mathbf{v}$. The kinetic energy is related to the total energy for both bradyons and tachyons via the relation $K = E - m_o c^2$.

In both SR and ER the force is defined to be the time rate of change of momentum, so that

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} = m\frac{d\mathbf{v}}{dt} + \mathbf{v}\frac{dm}{dt} \tag{35}$$

where m is the relativistic mass given by $m = \gamma_v m_o$ for bradyons and $m = \gamma_v m_*$ for tachyons. In the Newtonian limit $dm/dt \to 0$ and $m \to m_o$ so that $\mathbf{F} \to m_o(d\mathbf{v}/dt)$. The relativistic mass m is related to the total energy E by $E = mc^2$, and so

$$\frac{dm}{dt} = c^{-2}\frac{dE}{dt} = c^{-2}\frac{d}{dt}(K + m_o c^2) = \frac{\mathbf{F}.\mathbf{v}}{c^2}\,. \tag{36}$$

Substituting this result into (35) gives

$$\mathbf{F} = m\frac{d\mathbf{v}}{dt} + \frac{\mathbf{v}(\mathbf{F}.\mathbf{v})}{c^2}\,, \tag{37}$$

and as the acceleration in both SR and ER is defined to be $\mathbf{a} = d\mathbf{v}/dt$, then the acceleration is related to the force by

$$\mathbf{a} = \frac{\mathbf{F}}{m} - \frac{\mathbf{v}(\mathbf{F}.\mathbf{v})}{mc^2}\,. \tag{38}$$

This expression, applicable in both SR and ER, contains the standard 'Newtonian' term plus a purely relativistic correction term. The extra term has the direction of $\mathbf{v}$, so that $\mathbf{a}$ is not always parallel to $\mathbf{F}$.

There are two simple but interesting cases where $\mathbf{a}$ is parallel to $\mathbf{F}$: each of these cases relates to the motion of a charged particle in an external electromagnetic field.

Case (i): $\mathbf{F}$ is parallel to $\mathbf{v}$.

In this case $\mathbf{a}$ is parallel to both $\mathbf{F}$ and $\mathbf{v}$, so that using $m = \gamma_v m_*$ for tachyons in (35) gives

$$F = \gamma_v^3 m_* a\,. \tag{39}$$

The SR equivalent has m_* replaced by m_o. An example of this case is a charged bradyon or tachyon in a uniform electric field, with the magnetic field being zero and the particle's initial velocity being either zero or parallel to the electric field.

Case (ii): $\mathbf{F}$ is perpendicular to $\mathbf{v}$.

In this case $\mathbf{F}.\mathbf{v} = 0$ so that (38) reduces to

$$\mathbf{a} = \mathbf{F}(1 - v^2/c^2)^{1/2}/m_* \tag{40}$$

for tachyons. Once again, replace m_* with m_o to obtain the corresponding SR expression. An example of this case is that of a charged particle in a purely magnetic field, for which $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$. A charged bradyon travelling in a magnetic field traces out a circle with centripetal acceleration given by

$$\mathbf{a} = q(\mathbf{v} \times \mathbf{B})(1 - v^2/c^2)^{1/2}/m_o\,. \tag{41}$$

A charged tachyon travelling through the same magnetic field also traces out a circle, with real centripetal acceleration given by

$$\mathbf{a} = q(\mathbf{v} \times \mathbf{B})(1 - v^2/c^2)^{1/2}/m_*\,. \tag{42}$$

Therefore a high energy tachyon with speed approaching c describes a trajectory which is similar to the trajectory of a bradyon having the same magnitude of proper mass. This is appropriate because in the ultrarelativistic limit as $|\mathbf{v}| \to c$ the properties of bradyons and tachyons are virtually the same. The difference in trajectory between bradyons and tachyons becomes more pronounced as the energy decreases and the particle's speed becomes further removed from c. For bradyons the acceleration peaks at $v^2 = c^2/2$ but for tachyons the acceleration simply keeps increasing as $\mathbf{v}$ increases.

Tachyons and Cerenkov Radiation

One of the many objections raised with regard to tachyons is the notion that they emit Cerenkov radiation while travelling through a vacuum, as they are travelling faster than the speed of light in that medium (Jones 1972; Rockower 1975). Ey and Hurst (1977) have rigorously proved that tachyons will not emit Cerenkov radiation in a vacuum, but the case of a tachyon travelling through a medium with a refractive index greater than unity is far more complicated and requires careful analysis. An investigation of this problem has been carried out by Lemke (1975, 1976*a*, 1976*b*), while Recami and Mignani (1974) have shown that tachyons will not emit Cerenkov radiation in bradyonic media. The present formulation substantially agrees with the work of these authors, and so only a simple explanation for the lack of spontaneous tachyonic Cerenkov radiation in a vacuum needs to be given here.

Consider a vacuum in which there are two inertial observers Σ and Σ', with observer Σ' moving with constant speed $u > c$ relative to Σ. A particle travelling through that same vacuum appears to be a tachyon to observer Σ and a bradyon to observer Σ'. The observer Σ' notes that there are no forces, fields or potentials acting upon the particle, so that the particle has a constant velocity and does not undergo any acceleration whatsoever. The tachyonic transformations of velocity and acceleration derived earlier in this formulation indicate that a constant velocity in any inertial reference frame is still a constant velocity in any other inertial reference frame, regardless of whether those frames are bradyonic or tachyonic. As the particle will not spontaneously self-accelerate in a frame in which it appears to be a bradyon, it will therefore not do so when it appears to be a tachyon instead. Hence the particle will not emit Cerenkov radiation when it appears to be a tachyon travelling through a vacuum.

The same argument would apply if instead the particle appeared to observer Σ to be a bradyon and to observer Σ' a tachyon. It is known that in their own reference frames, such as frame Σ, bradyons do not spontaneously self-accelerate and emit Cerenkov radiation while travelling through a vacuum. Since particles which appear to be bradyons to observer Σ actually appear to be tachyons to observer Σ', it follows that Σ' will not detect any such Cerenkov radiation.

For a tachyon as seen from the laboratory frame, the vacuum appears to behave as if it has a velocity dependent refractive index which remains always greater than v/c. This conclusion holds for electromagnetic waves and charged tachyons but as Ey and Hurst (1977) have shown, it is also valid for gravitational waves and neutral tachyons. Hence gravitational Cerenkov radiation is not expected to be generated by free neutral tachyons.

5. Conclusion

The work presented here completes the treatment of tachyon mechanics and has demonstrated that this part of the physics of tachyons can be formulated in a logical and consistent manner. Mechanics by itself, however, provides no definitive proof as to whether or not tachyons exist. If there were no consistent framework for kinematics or dynamics then the concept of tachyons would not be worth pursuing, as in that case tachyons would be unable to interact with ordinary matter in a way that is detectable. One of the basic tenets of the current formulation is that tachyons, if they exist, must be able to interact with ordinary matter in a way that will be detectable. There is already in Section 4 an indication that important clues with regard to the existence of tachyons may come from a consideration of the interaction of tachyons and bradyons via the electromagnetic field, particularly if the plausible assumption is made that charged tachyons have the ability to generate a real electromagnetic field and a real Doppler effect.

A detailed discussion of tachyon electromagnetism will be the subject of the next paper in this series (Paper III), and will include a demonstration that tachyons are fully consistent with Maxwell's equations in their present form. The fourth paper in this series will develop electrodynamics for tachyons and will discuss the behaviour of charged tachyons in dielectric materials.

While tachyons will not emit Cerenkov radiation in a vacuum, it is not yet clear whether charged tachyons may or may not emit other forms of electromagnetic radiation when interacting with matter or fields. The two most interesting cases are bremsstrahlung and synchrotron radiation emitted in collisions between tachyons and in interaction with the magnetic field respectively. The present authors regard the investigation of those two forms of radiation as of paramount importance for the further study of tachyons. If such radiation can be emitted by tachyons then it should be detectable by bradyonic observers. Since little is known about the sources of this kind of radiation from astrophysical observations, one is tempted to conclude that tachyons, whether charged or not, are unable to radiate at all. Such a conjecture has already been made by Ey and Hurst (1977). However, if this is true then the tachyonic bremsstrahlung and synchrotron mechanisms would have to be destroyed by the transformation from the tachyonic rest frame to the laboratory (i.e. bradyonic) frame, especially since tachyons behave as bradyons in their own rest frame and will necessarily radiate. The investigation of this type of transformational destruction will be an important goal of the further study of tachyonic electrodynamics in this series of papers. It will be essential to calculate for the two types of radiation both the spectrum and the emission intensity and then to see what happens after transformation to the laboratory frame of reference.

The above comments underline the importance of electrodynamics for tachyon physics and in particular the question as to whether tachyons do in fact exist. These considerations, though, are entirely classical. It is to be expected that questions about the creation and annihilation of tachyons can only be answered within a quantum framework. This will follow only after a thorough discussion of the classical theory of tachyons has been completed.

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